Logistic Models

Math 130 - Essentials of Calculus

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Logistic Growth

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Sometimes a population increases exponentially at first, but then levels off when approaching its maximum population, called the *carrying capacity*, that the environmental conditions can sustain. The model for a situation like this is called a *logistic function* which has the form

$$P(t) = \frac{M}{1 + Ae^{-kt}}$$

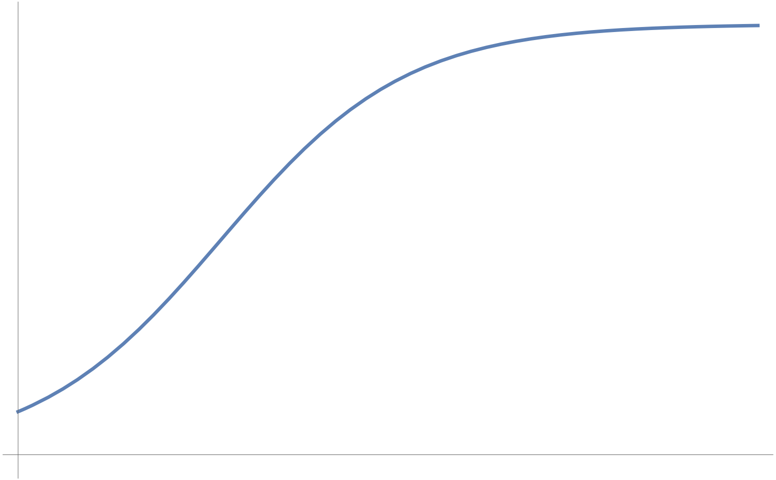
where M is the carrying capacity, t is time, k is a constant, and A is a constant given by

$$A = \frac{M - P_0}{P_0}$$

where P_0 is the *initial population*.

Logistic Growth

Here is the graph of a typical logistic function



Logistic Growth

Logistic growth also satisfies a differential equation given by

$$P'(t) = kP(t)\left(1 - \frac{P(t)}{M}\right).$$

The meaning of this equation is that the rate of change of the population is proportional to the product of the population and how far the population is from the carrying capacity.

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The meaning of this equation is that the rate of change of the population is proportional to the product of the population and how far the population is from the carrying capacity. Looking at the equation, we can see that $P'(t)$ is close to zero when $P(t)$ is close to zero, or $P(t)$ is close to M.

Example

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A lake is stocked with 1000 *fish and the fish population is expected to follow the model*

$$P(t) = \frac{17,000}{1 + 16e^{-0.7t}}$$

where t is the time elapsed, in years.

1. *What is the carrying capacity?*
2. *What is the fish population after* 2.5 *years?*
3. *How many years are required for the fish population to reach* 12,000*?*
4. *What is the growth rate of the fish population after five years?*

Example

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The number of mountain lions in a wildlife preserve is modeled by

$$P(t) = \frac{1680}{1 + 4.2e^{-0.11t}}$$

where t is the number of years after January 1, 2010.

1. *What is the carrying capacity? How many mountain lions are there on January 1, 2010?*
2. *According to the model, what is the population after 15 years?*
3. *When does the model predict that the mountain lion population will reach 1500?*
4. *Compute and interpret $P'(12)$.*